AP Calculus AB

Unit 6 – Area and Average Value

AP Calculus AB – Worksheet 51 **Approximating Area Using Riemann Sums**

Approximate areas "under the curve" (between the curve and the *x*-axis) using four subintervals for left, right and midpoint rectangles.

1. $f(x) = x^2$ on $[0,2]$
   a) Left Rectangular Approximation
   b) Right Rectangular Approximation
   c) Midpoint Rectangular Approximation

Approximate areas "under the curve" (between the curve and *x*-axis) using the indicated Riemann Sum.

2. $f(x) = x^3$ on $[0,2]$
   a) Find a Left Rectangular Approximation using four subintervals.
   b) Is the approximation found in part (a) an overestimate or underestimate? Explain how you know in terms of the increasing or decreasing behavior of the graph.

3. $f(x) = 1 + \cos x$ on $[0,\pi]$
   a) Find a Right Rectangular Approximation using four subintervals.
   b) Is the approximation found in part (a) an overestimate or underestimate? Explain how you know in terms of the increasing or decreasing behavior of the graph.

4. $f(x) = \sqrt{x}$ on $[0,4]$
   a) Find a Right Rectangular Approximation using four subintervals.
   b) Is the approximation found in part (a) an overestimate or underestimate? Explain how you know in terms of the increasing or decreasing behavior of the graph.

5. $f(x) = (x-1)^2$ on $[0,2]$
   a) Find a Midpoint Rectangular Approximation using four subintervals.

6. If $f(x) = \left(x^2 - 2x - 1\right)^{\frac{2}{3}}$, then $f'(0) =$

1	Approximate the area under $f(x)=(x-1)^2$ on $[0,4]$ using a) 4 rectangles whose height is given using the left endpoint b) 4 rectangles whose height is given using the right endpoint c) 4 rectangles whose height is given using the midpoint d) 4 trapezoids
2	Approximate the area under $f(x)=x^2+1$ on $[0,4]$ using a) 4 rectangles whose height is given using the left endpoint b) 4 rectangles whose height is given using the right endpoint c) 4 rectangles whose height is given using the midpoint d) 4 trapezoids
3	Approximate the area under $f(x)=(x+1)^2$ on $[0,4]$ using: a) 4 rectangles whose height is the left-hand endpoint b) 4 rectangles whose height is the right-hand endpoint c) 4 rectangles whose height is the midpoint of each subinterval d) 4 trapezoids
4	If $f(x)=\begin{cases} 2x & \text{for } x\le 1 \\ 3x^2-1 & \text{for } x>1 \end{cases}$, then find $\int_0^2 f(x)\,dx$ (Hint: split into two integrals)
5	$\lim\limits_{x\to 4}\dfrac{x^3-4x^2-x+4}{x-4}$
6	A function *f* that is continuous for all real numbers x has $f(3)=-1$ and $f(7)=1$. If $f(x)=0$ for exactly one value of *x*, then which of the following could be *x*? (A) -1 (B) 0 (C) 1 (D) 4 (E) 9
7	If $f(x)=\tan^2(8-2x)$, then $f'(1)=$

Answers:

1. a) 6 b) 14 c) 9 d) 10 exact: 28/3	2. a) 18 b) 34 c) 25 d) 26 exact: 76/3	3. a) 30 b) 54 c) 41 d) 42 exact: $\frac{124}{3}$	4. 7
5. 15	6. D	7. $-4\tan 6\sec^2 6$	

1. Consider the continuous function $f(x)$ such that $f(x)>0$ for $[0,1]$. Selected values of $f(x)$ are given in the table below. Use the table of values to approximate the area under $f(x)$ using the Riemann Sum indicated.

x	0	0.25	0.5	0.75	1.0
$f(x)$	1.0	0.8	1.3	1.1	1.6

a) Trapezoidal Approximation using 4 subintervals
b) Right Rectangular Approximation using 4 subintervals
c) Midpoint Rectangular Approximation using 2 subintervals

2. If a chart of values for the differentiable function $f(x) =$

x	0	4	16	17	20
$f(x)$	8	4	6	3	6

a) Find a trapezoidal approximation for the area under $f(x)$ on $[0,20]$ using four subintervals.
b) Find a right Riemann sum approximation for the area under $f(x)$ on $[0,20]$ using four subintervals.

3. $f(x)$ is a differentiable function that is increasing for all x. Selected values of $f(x)$ are given in the table below.

x	0	2	4	6	8	10
$f(x)$	12.5	13.4	13.9	14.3	14.6	14.8

a) Use a Left Rectangular Riemann sum to approximate the area under $f(x)$ on the interval $[0,10]$ using 5 subintervals of equal width. Is this approximation an underestimate or overestimate? Explain
b) Approximate $f'(5)$. Show the work that leads to your answer.
c) Find the average rate of change of $f(x)$ on the interval $[0,10]$.
d) Evaluate $\int_0^{10} f'(x)\,dx$

4. $R(t)$ is a differentiable function that is concave up for all t. Selected values of $R(t)$ are given in the table below.

t	0	3	5	9	11
$R(t)$	20	18	12	15	19

a) Use a trapezoidal sum to approximate the area under $f(x)$ on the interval $[0,10]$ using 4 subintervals. Is this approximation an underestimate or overestimate? Explain
b) Approximate $R'(4)$. Show the work that leads to your answer.
c) Find the average rate of change of $R(t)$ on the interval $[0,11]$.
d) Evaluate $\int_0^{11} \left(3+R'(t)\right)dt$

AP Calculus AB – Worksheet 54 **Exact Area Under a Curve**

Problems #1 – 4: Find the area under the graph of $f(x)$ from *a* to *b*. Graphing Calculator NOT permitted.

1. $f(x)=x+1; a=0, b=3$	2. $f(x)=4-x; a=-1, b=2$
3. $f(x)=4-x^2; a=-2, b=2$	4. $f(x)=4x-x^2; a=0, b=4$

Problems #5 – 8: Find the area under the graph of $f(x)$ from *a* to *b*. Graphing Calculator is permitted.

5. $f(x)=\cos x; a=-\frac{\pi}{2}, b=\frac{\pi}{2}$	6. $f(x)=\sin x; a=\frac{\pi}{6}, b=\frac{\pi}{3}$
7. $f(x)=e^{2x}; a=0, b=1$	8. $f(x)=e^{x}; a=-1, b=1$

<u>Graph</u> the region stated and then find the area of the bounded region.

9. Bounded by the curve $y=\sqrt{x}$ and the lines $x=4$ and $y=0$ (What is *a*? *b*?)

Express the limit as a definite integral. You do not need to evaluate the integral.

10. $\lim_{n\to\infty}\sum_{k=1}^{n} {x_k}^2\Delta x, \; [0,2]$	11. $\lim_{n\to\infty}\sum_{k=1}^{n}\left({x_k}^2-3x_k\right)\Delta x, \; [-7,5]$	12. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{x_k}\Delta x, \; [1,4]$

Use areas to evaluate each integral. Draw a sketch and shade in the appropriate regions.

13. Evaluate the integral: $\int_{-2}^{1} 5dx$	14. Evaluate the integral: $\int_{3}^{7}(-20)\,dx$	15. Evaluate the integral: $\int_{0}^{4} 3\theta\, d\theta$
16. $\int_{-2}^{4}\left(\frac{1}{2}x+3\right)dx$	17. $\int_{1/2}^{3/2}(-2x+4)\,dx$	18. $\int_{-3}^{3}\sqrt{9-x^2}\,dx$

Answers:

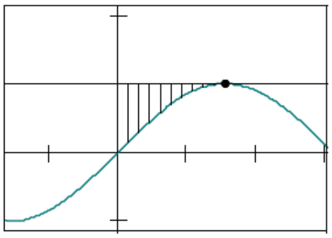
1. $\frac{15}{2}$	2. $\frac{21}{2}$	3. $\frac{32}{3}$	4. $\frac{32}{3}$
5. 2	6. $\frac{\sqrt{3}-1}{2}$	7. $\frac{e^2-1}{2}$	8. $\frac{e^2-1}{e}$
9. $a=0;\ b=4$ $\frac{16}{3}$	10. $\int_{0}^{2} x^2dx$	11. $\int_{-7}^{5}\left(x^2-3x\right)dx$	12. $\int_{1}^{4}\frac{1}{x}dx$
13. 15	14. -80	15. 24	16. 21
17. 2	18. $\frac{9\pi}{2}$		

These problems are a little trickier because the region bounded does not involve the *x*-axis.

For these problems, <u>you must</u>:

- **Graph the given functions to find the enclosed region that you will find the area of**
- **Write down: *Top function* - *Bottom function* (in terms of *x* only)**
- **Find the values for *a* and *b* (A little Algebra)**
- **Integrate to find area:** $\boxed{\text{Area} = \int_a^b \left(Top - Bottom\right) dx}$

1. Lying in the first quadrant and bounded by the curves $y = \sin x$, $y = 1$, and $x = 0$



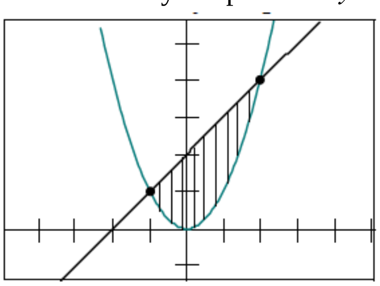
- What function is on *Top* of the shaded region? On the *Bottom*?

- What is *Top – Bottom?*

- What is *a*? *b*?

- Write the appropriate integral and find the area.

2. Bounded by the parabola $y = x^2$ and the line $y = x + 2$



- What function is on *Top* of the shaded region? On the *Bottom*?

- What is *Top – Bottom?*

- What is *a*? *b*?

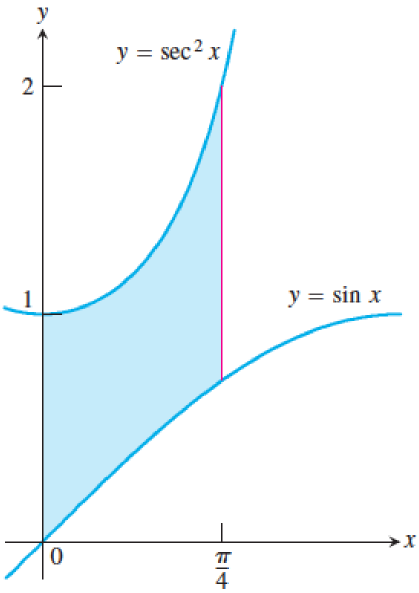
- Write the appropriate integral and find the area.

Find the total area between the curve and the *x*-axis in the given interval.

3) $y = 4 - x$ on $[0, 6]$	4) $y = \cos x$ on $[0, \pi]$

Find the area of the shaded region analytically

5)



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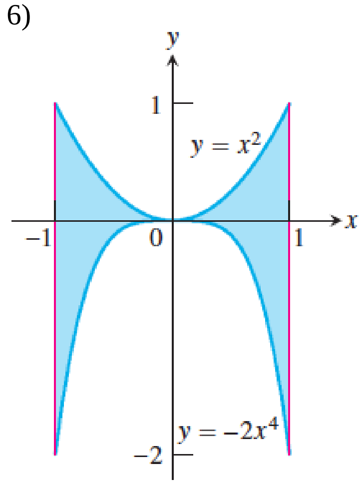

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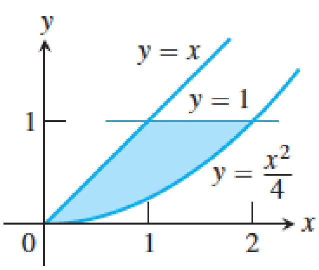

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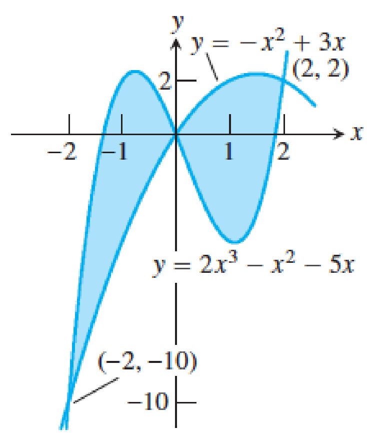


Find the area of the regions enclosed by the graphs of the curves. (Hint: find *a* and *b*)

9) $y = x^2 - 2$ and $y = 2$	10) $y = 7 - 2x^2$ and $y = x^2 + 4$

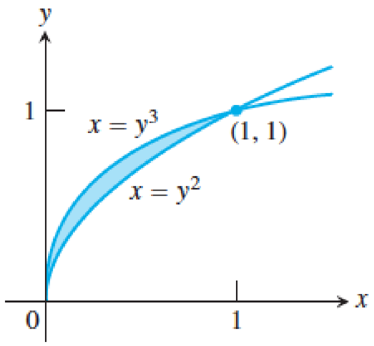
Answers:

3) 10	4) 2	5) $\frac{\sqrt{2}}{2}$	6) $\frac{22}{15}$
7) 5/6	8) 16	9) $10\frac{2}{3}$	10) 4

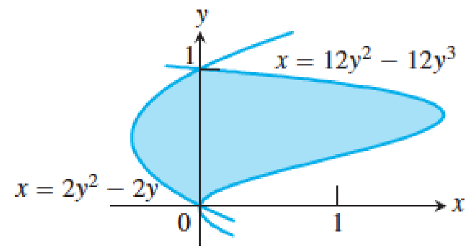
AP Calculus AB - Worksheet 56 **Area Between Two Curves – *y*-axis**

Find the area of the shaded region analytically.

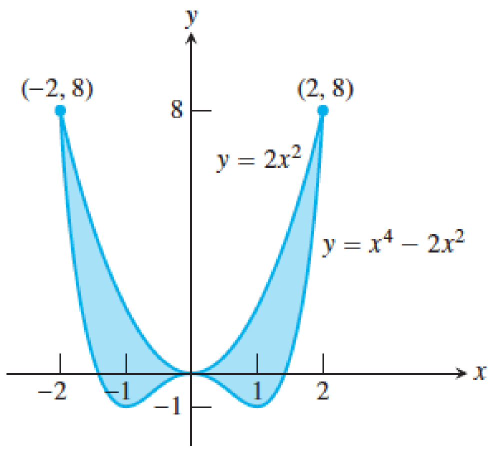
1)



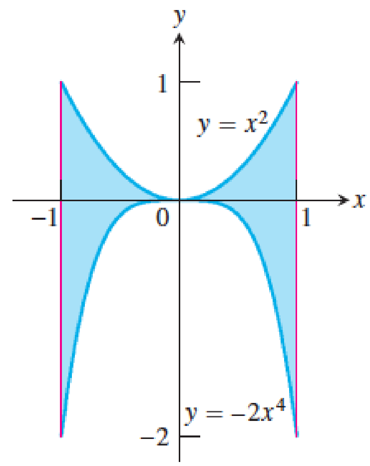
2)



3)



4)



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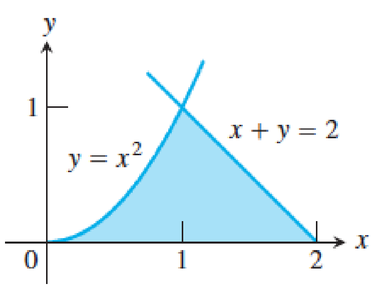
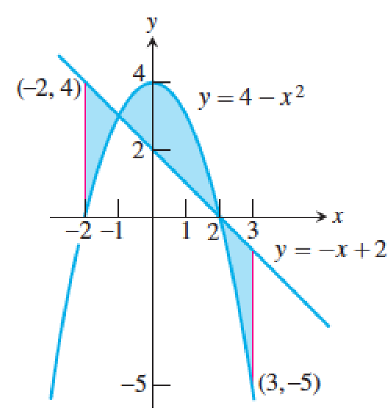


6)



7) Find the area of the region(s) enclosed by the graphs of $x - y^2 = 0$ and $x + 2y^2 = 3$.

8) Which of the following gives the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 3$?

A 2 **B** 9/2 **C** 13/2 **D** 13 **E** 27/2

Answers:

1) 1/12	2) 4/3	3) 128/15	4) 22/15	5) 5/6
6) 49/6	7) 4	8)		

AP Calculus AB - Worksheet 57 **Average Value and Review**

1.	**Graph** the region bounded by $y^2 = 4x$ and $y = 2x - 4$. Find the area
2.	**Graph** the region bounded by $y = 2 - x^2$ and $y = x - 4$ Find the area.
3.	**Graph** and find the area under the curve of $y = 2x + 1$ on $[0, 2]$.
4.	Find the average value of $f(x) = 5x^4 + 3x^2$ on the interval $-1 \le x \le 2$.
5.	Find the average value of $f(x) = \sin x$ on the interval [0, π].
6.	Find the average value of $f(x) = \frac{1}{x}$ on the interval [e, 2e].
7.	Find the average value of $y = 3x^2 + 2x$ on the interval $[-1, 2]$
8.	Find the average value of $y = \frac{1}{1+x^2}$ on the interval [0, 1].

Answers

1. 9	2. $\frac{125}{6}$	3. 6	4. 14
5. $\frac{2}{\pi}$	6. $\frac{\ln 2}{e}$	7. 4	8. $\frac{\pi}{4}$

1.	Find the average value of $f(x)=\sin x$ on the interval $\left[0,\frac{\pi}{4}\right]$.
2.	Approximate $\int_0^4 \left(x^2+2\right)dx$ using 4 subintervals by: a) Left-endpoint rectangles b) Right-endpoint rectangles c) Trapezoids d) Midpoint rectangles e) find the exact value of the integral
3.	Find the area between the following curves: $y=\sqrt[3]{x}$ and $y=x$.
4.	Calculate the area between the parabolas $y=25-x^2$ and $y=x^2-25$.
5.	Find the value of k such that the following function is continuous for all real numbers. $f(x)=\begin{cases} kx-1, & x<2 \\ kx^2, & x\geq 2 \end{cases}$
6.	Find the area of the region enclosed by the graphs of $y=x^2$ and $y=2x+3$.
7.	Find the area between the curve $y=\sin 3x$ and the x-axis from $x=0$ to $x=\frac{\pi}{3}$.
8.	Write, but do not evaluate, the integral expression that can be used to find the area between the curves of $x+2=y^2$ and $y=x$.
9.	For what value of c is $f(x)=\begin{cases} 3x^2+2, & x\geq -1 \\ -cx+5, & x<-1 \end{cases}$ continuous?
10.	Find the average value of $y=\sec^2 x$ on $\left[0,\frac{\pi}{4}\right]$.

Answers

1. $\left(-\frac{\sqrt{2}}{2}+1\right)\frac{4}{\pi}$	2. 22; 38; 30; 29; $\frac{88}{3}$	3. $\frac{1}{2}$	4. $\frac{1000}{3}$	5. $-\frac{1}{2}$
6. $\frac{32}{3}$	7. $\frac{2}{3}$	8. $\int_{-1}^{2}\left[y-\left(y^2-2\right)\right]dy$	9. 0	10. $\frac{4}{\pi}$